Master in Actuarial Science

Risk Theory

31-05-2011

Time allowed: Three hours

Instructions:

1. This paper contains 8 questions and comprises 3 pages including the title page.
2. Enter all requested details on the cover sheet.
3. You have 10 minutes of reading time. You must not start writing your answers until instructed to do so.
4. Number the pages of the paper where you are going to write your answers.
5. Attempt all 8 questions.
6. Begin your answer to each of the 8 questions on a new page.
7. Marks are shown in brackets. Total marks: 200.
8. Show calculations where appropriate.
9. An approved calculator may be used.
10. The distributed Formulary and the Formulae and Tables for Actuarial Examinations (the 2002 edition) may be used. Note that the parametrization used for the different distributions is that of the distributed Formulary.

1. The number of claims of a risk are reported according to a Poisson process with rate = 0.5 per year. Claim sizes are i.i.d. random variables, independent of the number of reported claims. The amount of each claim is Pareto with mean 1 hundred thousand euros and standard deviation $\sqrt{3}$ hundred thousand euros. The insurer reports to the reinsurer every claim exceeding 8 hundred thousand euros.

   Let W_n denote the reporting time of the n-th claim and S_n the reporting time of the $n-$th reinsured claim.

   (a) Calculate the probability that W_2 happens before 5 years have elapsed. (10)

   (b) Calculate the probability that W_2 happens after 5 years and before 10 years have elapsed. (10)

   (c) Calculate the probability that S_1 happens before 10 years have elapsed. (10)

2. Consider that the number of accidents per year, N, of a policy, follows a negative binomial distribution with parameters r and β. Suppose that each time that there is an accident, the policyholder reports a claim if the amount of the loss exceeds the deductible d. Let $F_X(x)$ be the distribution function of the losses, and suppose that they are independent, and independent of the number of accidents. Let M be the number of reported claims per year.

   (a) Writing M as a compound mixed Poisson distribution, and using generating functions show that the distribution of M is a negative binomial with parameters r and $\beta(1 - F_X(d))$. (15)

   (b) Considering $r = 2.5$, $\beta = 1$, $d = 2$ and that X is Gamma distributed with parameters $\alpha = 2$ and $\theta = 1$, calculate the probability that M is 2. (10)

3. The number of claims of a risk is modeled by a mixed Poisson process with structure random variable Λ, which has distribution function $U(\lambda) = \Pr\{\Lambda \leq \lambda\}$. Let $p_k(t) = \Pr\{N(t) = k\}$, $k = 0, 1, 2, ...$ and $p_{k,k+n} = \Pr\{N(t) - N(s) = n | N(s) = k\}$ for $k, n = 0, 1, 2, ...$ for all $0 < s < t$.

   (a) Determine $p_k(t)$, $k = 0, 1, 2...$ (10)

   (b) Prove that $p_{k,k+n}(s,t) = \binom{k+n}{n} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^n \frac{p_{k+n}(t)}{p_k(s)}$. (20)

4. The number of claims in a period has a geometric distribution with mean 5. The amount of each claim X follows a discrete uniform on the set $\{0, 1, 2, 3, 4\}$. The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period. Calculate $F_S(3)$. (25)

5. Claim size X has a distribution with survival function $S_X(x) = \left(\frac{\theta}{x+\theta}\right)^2$, for $x > 0$. You are given that $VaR_{0.99}(X) = 900$. Calculate $E[X \wedge 50]$, the expected value of the claim size subject to a limit of 50. (10)

6. Let the aggregate claims from a risk be distributed according to a Pareto with parameters (α, θ), with $\alpha > 1$ and $\theta > 0$. The insurer arranges stop-loss reinsurance with retention limit M and unlimited cover. Both the insurer and the reinsurer calculate their premiums using the PH transform premium principle with (the same) parameter ρ, where $1 \leq \rho < \alpha$.

   Derive formulae in terms of α, θ, ρ and M for:

   (a) The insurer's premium (before taking account of reinsurance). (10)

   (b) The reinsurer's premium. (10)

7. Consider that the aggregate claims of a portfolio follow a compound Poisson process, and that the density function of the individual claim size is

$$f_X(x) = \frac{9}{25} x \mathrm{e}^{-3x/5}, \quad x > 0.$$

   Let the premium be calculated according to the variance principle with loading coefficient 0.1.

   (a) Show that the premium is 5λ, where λ is the rate of the Poisson process. (10)

   (b) Calculate $\psi(0)$. (10)

   (c) Calculate the adjustment coefficient. (15)

8. Let the annual number of claims of a given portfolio be Poisson distributed with mean 1000 and let the logarithm of the individual claim size be Normal distributed with mean 2 and standard deviation 1. Using the NP approximation, calculate the total premium so that the ratio between the aggregate claims and the total premium is smaller than 105%, with a probability of 99%. (25)